Function Operations

Examples

1. Given $f(x) = 3x + 4$ and $g(x) = x^2 - 5x + 2$, find each part below and the domain for each.

a) $(f + g)(x)$

$3x+4 + x^2-5x+2$

$x^2 - 2x + 6$

Domain: all real #'s (ARN)

b) $(g - f)(x)$

$x^2 - 5x + 2 - (3x+4)$

$x^2 - 5x + 2 - 3x - 4$

$x^2 - 8x - 2$

Domain: ARN

c) $(f \cdot g)(x)$

$(3x+4)(x^2 - 5x + 2)$

$3x^3 - 15x^2 + 6x$
$+ 4x^2 - 20x + 8$

$3x^3 - 11x^2 - 14x + 8$

Domain ARN

2. Given $f(x) = 3 - 2x$ and $g(x) = 2x^2 + 7x - 1$, find each part below and the domain for each.

a) $(f + g)(x)$

$3 - 2x + 2x^2 + 7x - 1$

$2x^2 + 5x + 2$

Domain: ARN

b) $(g - f)(x)$

$2x^2 + 7x - 1 - (3 - 2x)$

$2x^2 + 7x - 1 - 3 + 2x$

$2x^2 + 9x - 4$

Domain: ARN

c) $(f \cdot g)(x)$

$(3 - 2x)(2x^2 + 7x - 1)$

$6x^2 + 21x - 3$
$-14x^2 + 2x \qquad -4x^3$

$-4x^3 - 8x^2 + 23x - 3$

Domain: ARN

3. Given $f(x) = 3x + 4$ and $g(x) = x^2 - 5x + 2$, find each part below and the domain for each.

a) $(f + g)(2)$

$f(2) + g(2) = \boxed{6}$

$f(2) = 3(2) + 4 = \underline{10}$

$g(2) = 2^2 - 5(2) + 2 = -4$
$\qquad\quad 4 - 10 + 2$

b) $(g - f)(-1)$

$g(-1) - f(-1) = 8 - 1 = \boxed{7}$

$g(-1) = (-1)^2 - 5(-1) + 2$
$\qquad\quad 1 + 5 + 2 = 8$

$f(-1) = 3(-1) + 4 = \underline{1}$

c) $(f \cdot g)(1)$

$f(1) \cdot g(1) = 7 \cdot -2 = \boxed{-14}$

$f(1) = 3(1) + 4 = 7$

$g(1) = 1^2 - 5(1) + 2$
$\qquad\quad 1 - 5 + 2 = -2$

4. Given $f(x) = x - 7$ and $g(x) = 2x^2 - 13x - 7$, find $\left(\dfrac{f}{g}\right)(x)$ and the domain of the quotient.

$\dfrac{f(x)}{g(x)} = \dfrac{x - 7}{2x^2 - 13x - 7} = \dfrac{\cancel{x-7}}{(2x+1)\cancel{(x-7)}} = \boxed{\dfrac{1}{2x+1}}$

Domain: ARN except $x = 7$
(before cancelling) & $x = -\frac{1}{2}$

$\left(\begin{array}{c} 2x+1 = 0 \\ 2x = -1 \\ x = -\frac{1}{2} \end{array}\right)$

5. Given $f(x) = x + 3$ and $g(x) = x^2 - 3x - 18$, find $\left(\frac{g}{f}\right)(x)$ and the domain of the quotient.

$$\frac{g(x)}{f(x)} = \frac{x^2 - 3x - 18}{x+3} = \frac{(x-6)(x+3)}{(x+3)} = \boxed{x-6}$$

$\boxed{\text{Domain: ARN except } x = -3 \text{ (before cancelling)}}$

6. Given $f(x) = x^2$ and $g(x) = x + 1$, find $f(g(3))$

$$g(3) = 3 + 1 = 4$$
$$f(g(3)) = f(4) = \boxed{16}$$

7. Given $f(x) = x^2$ and $g(x) = x + 1$, find $f(g(x))$

$$f(g(x)) = f(x+1) = (x+1)^2 = (x+1)(x+1)$$
$$= x^2 + x + x + 1$$
$$= \boxed{x^2 + 2x + 1}$$

8. Given $f(x) = 2x - 1$ and $g(x) = 3x$, find $f(g(2))$

$$g(2) = 3(2) = 6$$
$$f(g(2)) = f(6) = 2(6) - 1 = \boxed{11}$$

9. Given $f(x) = 4x^2 + 7$ and $g(x) = 6x + 9$, find $f(g(x))$ and $g(f(x))$

$$f(g(x)) = f(6x+9) = 4(6x+9)^2 + 7$$
$$= 4(6x+9)(6x+9) + 7 = 4(36x^2 + 54x + 54x + 81) + 7$$
$$= 4(36x^2 + 108x + 81) + 7$$
$$144x^2 + 432x + 324 + 7$$
$$f(g(x)) = \boxed{144x^2 + 432x + 331}$$

$$g(f(x)) = g(4x^2 + 7)$$
$$= 6(4x^2 + 7) + 9$$
$$= 24x^2 + 42 + 9$$
$$= \boxed{24x^2 + 51}$$